

nia pacano a spinster; pacanoka my second child (hijo, hija de en medio). viro pacano unmarried man.
 pequata vassal.
 pona to come; ni ponala I come.
 ponachica? do you bring? (see: viro).
 soba meat; deer-meat.
 taca fire; coal-fire; taca chale new coal-fire (candela nueva in Span.); taca chu carbon, charcoal.
 tafi sister-in-law (used by men only); tafimitana, ni tafimitama the brother of my husband (used by women).
 tico canoe, boat; ticopaha ship.
 tinibo woodpecker.
 ulipassa fragments of pottery (uli = Span. olla).
 uti earth, land, country; uti-hasomi those forming the Earth-pedigree or Earth-Family, utina my country; uti nocoromale the inhabitants of ONE country.
 viro man; chiri viro boy, son; viroleqe uquata puenonica, I brought here a male child.

The Approaches to a Theory of the Cause of Magnetic Declination.

BY PERSIFOR FRAZER, JR.

(*Read before the American Philosophical Society, April 6, 1877.*)

So many questions of historical, economical and scientific interest are bound up with the variation of the magnetic needle from the astronomical North, and the change in the rate and sign of this variation, that the following remarks may be excused, even if they only succeed in impressing upon the imagination the immense amount of work which yet remains to be done. The best compendium of the history of the subject is contained in a prize essay on Terrestrial and Cosmical Magnetism by E. Walker, Cambridge (Eng.), 1866, while for particular discussions of special groups of observations, Sabine's Secular Variations of the Magnetic Needle, in the Trans. of the Royal Society during the last five years; Prof. A. D. Bache's discussion of the magnetic elements, observed at Girard College during the years 1840-45 inclusive; in Coast Survey Reports for 1855, 1858, 1859, 1860 and 1862, and especially Mr. C. A. Schott's labors on these and all other obtainable data in the United States, have been mainly drawn upon. Besides these, for general questions relating to the subject, Barlow's treatise in the Encyclopædia Metropolitana; Airy's treatise on Magnetism, London, 1870; Numerical Relations of Gravity and Magnetism, by P. E. Chase, Trans. A. P. S. 1864; Prof. Loomis' collection of magnetic observations; Silliman's Journal, 1838 to 1840; Reclus, Despretz, Becquerel, Humboldt, "Magnetism," by Sir W.

S. Harris and H. M. Noad, and "Diurnal Variations of Declination," by Wm. Norton, have been consulted.

A large number of observations with their authorities will be found at the end of the Report of C. A. Schott in C. S. Report for 1858.

Resumé from Coast Survey Reports.

In the C. S. Report for 1855, p. 302, is a chapter by Prof. A. D. Bache, on the Magnetic Declination in the northern part of the Gulf of Mexico. The observed declinations are reduced to the common date of Jan. 1850, by an assumed annual decrease of $0'.5$ E. declination from observations made at Pascagoula in 1847 and 1855.

The equation representing the true declination with reference to the latitude L and the longitude M of any place, here follows :

$$dV = v + x dL + y dM + z dL dM + p dL^2 + q dM^2$$

dV = difference between the observed decl. and the assumed decl. V'

V = correction to the assumed V' .

The solution of the conditional equation, for any latitude L and longitude M gives the following expression for V :

$$dL = L - 28^\circ .04. \quad dM = M - 88^\circ .69.$$

$$V = 7^\circ .39 \text{ East} - 0.025 dL + 0.296 dM + 0.0188 dL dM - 0.0094 dL^2 - 0.0076 dM^2.$$

All the declinations being East are treated as essentially positive quantities.

The number of groups of stations selected was six, in order to solve the six unknown quantities in the first equation.

There were the following number of stations in each group. The Roman numeral indicates the number of the group, and the Arabic numeral the number of stations : I, 1 ; II, 1 ; III, 4 ; IV, 4 ; V, 3 ; VI, 1.

The average of the residual—(*i. e.* the difference between the observed and computed declinations)—was $0^\circ 056 = 3'.3$.

For the observations in the general table, the form of the conditional equation is the same as above.

$$V = V' + v + x X + y Y + z X Y + p X^2 + q Y^2.$$

The annual variations were an estimated approximation by mutual comparison of the known values at Toronto and on the Atlantic coast.

The origin of co-ordinates is assumed midway between lat. $43^\circ 26'$, long. $70^\circ 24'$; and lat. $29^\circ 07'$, long. $83^\circ 03'$; the co-ordinates of position are expressed in degrees and decimals and *were graphically obtained* by careful plotting on a large scale.

Mr. Chas. A. Schott in the same volume furnishes a discussion of the magnetic declination on the Atlantic and part of the Gulf coast (communication to the American Association for the Advancement of Science, Appendix 48 to Coast Survey Report for 1855,) of which the following are some of the points :

Hausteen's publication "Investigations of the secular variation since the magnetic observations in Coast Survey Report of 1854," gave a new impulse to these investigations.

Dr. Bowditch and Prof. Loomis were mainly instrumental in bringing this subject prominently to the notice of scientific men.

Prof. Loomis gives (Silliman's Journ. of Sci. and Arts 1840) the annual change of variation in 1840 as 2' for the Southern States, 4' for the Middle and 6' for the New England States.

The éssay is divided into : (a) Stations of reliable observations prior to 1740 ; (b) Ditto after that time ; (c) Results from comparatively recent observations ; (d) Establishment of formulæ expressing the secular variation within the limits indicated on the title page (*i. e.* the Atlantic and part of the Gulf coast).

Under (a), Providence, R. I., Hatboro', and Philadelphia, Penna., are selected. Thirty declinations collected by Mr. M. B. Lockwood from actual observations and recorded bearings of a number of objects, are considered. The formula employed is

$$D = d_0 + y (t - t_0) + z (t - t_0)^2 + u (t - t_0)^3 + \&c.$$

Where y, z, u are unknown coefficients, and $D = d$ where $t = t_0$. Putting $d_0 = d_1 + x$ where x is a small correction to the assumed value of d , and omitting higher powers of the time, than the third

$$D = d_1 + x + y (t - t_0) + z (t - t_0)^2 + u (t - t_0)^3.$$

Assuming for t_0 the commencement of any year and for d_1 the supposed corresponding declination (expressed in degrees and decimals) then each observed value for D at the time t furnishes the following conditional equation : $0 = d_1 - D + x + y (t - t_0) + z (t - t_0)^2 + u (t - t_0)^3$.

From this by the application of least squares the normal equations and coefficients of x, y, z, u , are obtained.

"The above formula is capable of giving two maxima and two minima, whereas the omission of the third power would give but a minimum. *And this as we know from observation took place about the commencement of this century.* t_0 is assumed as 1830 ;* and $d_1 = + 7.20$."

A table of thirty observations in Providence, R. I., is reduced in size by substituting a table of sixteen means, which here follow :

1717	D = + 9.60		1790	D = + 6.18
1720	9.47		1800	6 .25
1730	8.92		1810	6 .40
1740	8.28		1820	6 .66
1750	7.67		1830	7 .19
1760	6.99		1840	8 .42
1770	6.49		1842	8 .65
1780	6.27		1844-8	9 .25

Substituting in the above equations we have :

$$0 = - 2.40 + x - 113 y + 12769 z - 1442897 u,$$

and similarly for the other fifteen equations.

Multiplying each of the sixteen equations by the coefficient of the first unknown quantity and adding them all up, we get the first of the normal equations, and the same operation performed for each unknown quantity will give the other normal equations.

* In the discussion of subsequent years this date was changed to 1850.

These would be as follows :

$$\begin{aligned}
 0 &= - 7.49 + 16 x - 736.2 y + 63832 z - 5792927 u \\
 0 &= + 573.26 - 736.2 x + 63832 y + 5792927 z - 562891700 u \\
 0 &= - 79077 + 63832 x - 5792927 y + 562891700 z - 56610893300 u \\
 0 &= + 8782402 - 5792927 x - 562891700 y - 56610893300 z + 5831932000000 u
 \end{aligned}$$

To facilitate reduction Mr. Schott assumes $x = X, y = Y : 10^2, z = Z : 10^4, u = U : 10^6$. Dividing the first equation by 10^0 , the second by 10^2 , the third by 10^4 , and the fourth by 10^6 , he obtains the modified normal equations.

$$\begin{aligned}
 0 &= - 7.4900 + 16.0000 X - 7.3620 Y + 6.3832 Z - 5.7929 U \\
 0 &= + 5.7326 - 7.3620 X + 6.3832 Y - 5.7929 Z + 5.6289 U \\
 0 &= - 7.9077 + 6.3832 X - 5.7929 Y + 5.6289 Z - 5.6611 U \\
 0 &= + 8.7824 - 5.7929 X + 5.6289 Y - 5.6611 Z - 5.8319 U
 \end{aligned}$$

Their solution gives

$$\begin{array}{ll}
 X = + 0.239 & \text{hence } x = + 0.239 \\
 Y = + 8.543 & y = + 0.08543 \\
 Z = + 15.055 & z = + 0.0015055 \\
 U = + 5.100 & u = + 0.000005100
 \end{array}$$

and the formula for the declination becomes

$$D = + 7^{\circ}.439 + 0.08543 (t - 1830) + 0.0015055 (t - 1830)^2 + 0.000005100 (t - 1830)^3.$$

A Table of Comparisons of the Observed and Computed Declinations at Providence here follows :

DATE.	OBSERVED.	COMPUTED.	Δ	Δ^2
1717.....	+9.60	+9.64	+0.04	0.0016
1720.....	9.47	9.46	-0.01	0.0001
1730*.....	8.92	8.85	-0.07	0.0049
1740.....	8.28	8.22	-0.05	0.0025
1750.....	7.67	7.62	+0.05	0.0025
1760.....	6.99	7.08	+0.09	0.0081
1770.....	6.49	6.63	+0.14	0.0196
1780.....	6.27	6.29	+0.02	0.0004
1790.....	6.18	6.10	-0.08	0.0064
1800.....	6.25	6.09	-0.16	0.0256
1810.....	6.40	6.29	-0.11	0.0121
1820.....	6.66	6.73	+0.07	0.0049
1830.....	7.19	7.44	+0.25	0.0625
1840.....	8.42	8.45	+0.03	0.0009
1842.....	8.65	8.69	+0.04	0.0016
1844-8.....	9.25	9.05	-0.20	0.0400

$$\epsilon_0 = 0.674 \sqrt{\frac{y \Delta^2}{n-4}} = \pm 0^{\circ}.085 = \pm 5'.1$$

Mr. Schott proceeds to the establishment of formulæ expressing the sec

* A misprint makes this 1720 in Appendix No. 48, Coast Survey Report for 1855

ular variation in the rate of magnetic declination at any place within the limits of the stations named in the discussion.

By the tables thus framed the average error in the computed declinations is ± 0.0063 .

He adds that the small table extracted from the Phil. Trans. Royal Soc. Vol. XI, abridged, from 1755 to 1763, which Hausteen used in the construction of the isogonic curves in his Erdmagnetismus, and which here follow are erroneous, inasmuch as we know that the Western declination had been decreasing from 1700 to about 1797, while the last two lines of this table make it seem to increase. It may be remarked that the first two lines virtually show increase of Westerly, because they show decrease of Easterly declination.

OBSERVATIONS BY P. F. JR.	DECLINATIONS IN					
	LAT.	LONG.	1700.	1730.	1744.	1756.
30 miles E. of Rodriguez Key, Fla.....	25°	80°	4 $\frac{3}{4}$ E.	—	3 $\frac{1}{2}$ E.	3 E.
76 miles E. 8° N. of St. Augustine, Fla.....	30°	80°	2 $\frac{1}{2}$ E.	—	3 $\frac{3}{4}$ E.	0
32 miles E. S. E. of Cape Hatteras.....	35°	75°	2 $\frac{1}{4}$ W.	—	6 $\frac{1}{2}$ W.	7 W.
365 miles E. of Sandy Hook.	40°	70°	7 W.	9 W.	11 $\frac{1}{2}$ W.	12 $\frac{3}{4}$ W.

Mr. Schott's contribution to our knowledge concludes with the following remark :

“ Before closing the appendix it will be seen that a look out for a time of inflexion to be expected about 1867 \pm 15 years was not premature and indeed we find from the following comparison of the computed, and my last observed declinations that the latter always fall short of the former—a plain indication that the curve commenced turning its concave side towards the axis of abscissæ ; or in other words that in 1855.5 the maximum annual increase had already been passed.”

The annual variation of 1850 is recommended as a constant for some years till a new series clears up the point. He adds : * “ Thus for example the declination in Boston in 1870 will be found by computing the increase for twenty years (the difference of 1870 and 1850) prior to 1850, and adding the same to the declination for 1880 ; the declination at Boston for 1870 becomes $+ 9^{\circ}.81 + 1^{\circ}.48 = 11^{\circ} 18' W.$ ” †

Referring to Mr Schott's admirable *announced tables* to accompany the C. S. Report for 1874 we find the Declination at Boston for 1870 given at $10^{\circ}.90$ (see table of decennial values). This varies from his prophecy by $0^{\circ}.28$ or $16'.8$ (or with the correction suggested below by $0^{\circ}.39$ or $23'.4$).

* Written in 1855.

† Should this not be $11^{\circ}.29$?

He concludes by calling attention to the following epochs.

	<i>Differences.</i>
Deduced maximum declination in 1679 ± 10 years.....	} 62 years.
Known first point of inflexion 1741 ± 10 years	} 57 years.
Known minimum declination 1798 ± 2 years	} 52 years.
Supposed second point of inflexion 1850.	

“From which it appears that the periods are diminishing or the velocity of the secular variation is increasing, which latter is sustained by a comparison of $V_{1741} = -4'.6$ with $V_{1850} = +6'.8$ or $+5'.9$,” &c.

In Coast Survey Report for 1858 p. 192 to 197 Mr. Schott resumes the discussion of the subject. He announces the discovery at Hatboro, Pa. of a longer period of 234 years and a shorter one of 88 years, the range of the secondary motion being about $\frac{1}{15}$ of that of the primary or about $0^{\circ}.25$. The length of the shorter period as well as its epoch and range is different in different localities, but the fact of the existence of two such periods was afterwards confirmed by discussions of the periods at Burlington Vt. and Providence, R. I.

For the representation of the Hatboro observations the form was employed: $D = d + r \cos. (an + c) + r_1 \cos. (a, n + c_1)$ when $n =$ number of years $\left. \begin{array}{l} + \text{ after} \\ - \text{ before} \end{array} \right\}$ an assumed epoch; in this case the year 1830.

In the numerical application, the last term being neglected, the form adopted for the conditional equations was :

$$0 = -D + d_1 + x + \cos. a n r \cos. c - \sin. a n r \sin. c.$$

The first assumption for d_1 was $= 5.2$ and $a = \frac{3.60}{250} = 1.44$ as pointed out in discussion of 1855.

In a second and third assumption (a) was varied. Afterwards that value was assumed which made the sum of the squares of existing differences a minimum. The probable ϵ_0 was $\pm 8'.6$ as against $\pm 11'.0$ in the former discussion.

The condition of the minimum declination is expressed by the formula $0 = 5.05 \sin. (1.54n + 46^{\circ}.8) - 0.90 \sin. (4^{\circ}.1 n - 13^{\circ})$ from which $n = -33.7$ years. Hence the minimum occurred in 1796.3.

The effect of the last term is to place the minimum 3.3 years earlier. The former discussion (in 1855) placed the minimum at 1806.1 ± 19.3 years and the mean for all stations then discussed gave $1797.6 + 1.8$ years.

By a first and second differentiation of the above formula $v = +45.2$ or *the maximum annual change will occur in 1875.2.*

From the observations since 1750, separately discussed in 1855 $T = 1799.5$.

From the next following discussion of the magnetic declination at Washington, it is concluded that the maximum declination at the close of the last century was $+0^{\circ}.42'$ or *the line of no variation* at its highest.

ascent at that period *passed below Washington. It certainly passed above Norfolk.* *

The maximum declination will probably be = 40.42 .

In Mr. Schott's discussion of the secular change in C. S. Report for 1859 he mentions that no entire cycle has yet been completed on either the East or West coast. The linear form of the formula first applied to all observations in 1855 (which does not involve any great length of period) is retained for the observations on the Western coast while a circular function is chosen for the others, of which the length of period and other numerical co-efficients have been obtained by the method of least squares. But he adds, "*as long as the cause producing the secular change remains altogether unknown it is not safe to trust too far to the continuation of the law thus empirically derived.*"

He finds that "if the stations be arranged geographically, the minimum (West) declination occurred earlier at the Eastern than at the Western and Southern stations." * * * "*and if we proceed to the Western coast we find that the Eastern declination has not yet reached its maximum (equivalent to a Western minimum.)*"

This Report concludes with a record of all declination observations employed in the forgoing papers.

From the preceding imperfect memorandum of the analytical methods of some of the ablest mathematicians and magneticians, who have taken this subject in hand, and especially of those of Mr. Schott, it will be seen :

1st That to the best informed the cause of variation in declination remains still unknown.

2d The mathematical analysis proves the existence of a period including secondary periods, the latter resulting from perturbations due to some cause not identical with the main cause.

3d Over small areas and during short intervals of time the magnetic variation can be predicted and a formula established which shall express it.

4th The linear formula having shown the change of the *kind* of variation in abscissa after a certain epoch and in the *kind* of variation in the ordinate after another epoch establishes the cyclic curve as one of unequal axes.

5th. The easternmost North and South line which is tangent to this curve passes nearly through Washington and some distance west of Philadelphia. The westernmost North and South tangent is not yet determinable with exactness.

* By reference to the Isogonic chart previously mentioned, kindly corrected and sent me by Mr. Schott, March 17, 1876, the highest position attained by the Zero curve is marked in red ink as above Annapolis and Baltimore in 1795, while by computing the mean annual declination from the tables from Report of 1874 (Philadelphia) it will be found that the maximum was from 1740 to 1760 when it amounted to 107.8 , and the minimum from 1800 to 1810 when it amounted to $0'.0$

The column for Washington only goes back to 1800 but its minimum seems to have been between 1790 and 1800. From this to the present time this annual mean for Washington presents several irregularities.

6th. The situation of this curve cannot be very near the Terrestrial Pole, since on the same parallel of latitude and with a distance apart of about 2,685 miles, Passamaquoddy Bay shows a (West) declination of $+ 18^\circ$ and Salem, Oregon, an Easterly declination of $- 20^\circ$ for the same year. These curves unless forming abnormally acute cusps must meet if produced below lat. 70° as drawn in Col. Sabine's chart.

7th. It is probable that the location of the area of magnetic attraction is nearer to the Atlantic than to the Pacific, because the Isogonic curves of even degrees are nearer together in the former region than in the latter.

8th. Any theory which may be established to account for secular variation must accept the end of the last century or the beginning of this as one of the extremes.

9th. The total period of revolution according to the best data is about 237* years with a margin of a few years error more or less.

10th. The limits within which the declination for a particular date may be calculated for a station, where reliable observations are at hand, is about $8'$ to $11'$.

The following table was calculated to represent the magnetic variation at Mount Holly, Cumberland Co., Penna., for every five years since 1790, and may serve as an example of how such approximately accurate tables may be obtained for points where the records are very meagre.

In the first place the Isogonic chart of the Coast Survey for 1870 (as corrected by the pen of Mr. Chas. A. Schott) was referred to for the positions of various isogonic curves which passed at different dates in the vicinity of Mount Holly. The exceedingly small scale of this chart and the uncertainty of the data from which many of the curves were traced, renders the attainment of the required declination only possible within rather wide limits, amounting perhaps to $20'$ if all errors be taken into account; or a little more than the minimum (quarter of a degree) noticed by an ordinary surveyor.

Of course, if the chart be assumed as correct, the error is reduced to less than half this. The method employed for determining these declinations is as follows: After locating the position of the place whose declination is sought on the chart by latitude and longitude, a normal to the isogonic curves of the date required is drawn through this place. The distance apart on this line of the nearest curves of whole degrees between which the place lies, is accurately measured, and afterwards the distance from the point to that one of these curves which is represented by the smallest number of degrees. This distance in the same units multiplied by sixty and divided by the former number will give the number of minutes, which being added to the smallest number of degrees represented by the two curves, furnishes the correct declination of the place at the date for which the curves are drawn. If any actual observations of the declination at the desired locality are at hand these may be compared with the nearest place at which observations have been carried on, and the annual rate of change of the

* See Mr. Schott's Report, *ante*.

amount expressing the difference between the two declinations calculated and noted for each term of years between such data. When observations are entirely wanting, an approximation to the true declination may be made in the manner already described; though the accuracy of such a method will be inversely proportional to the distance along the isogonic normals to old stations of record, and directly as the distance apart of adjacent curves.

DATE.	DECLINATION AT		DIFFERENCE.	REMARKS.
	PHILADELPHIA.	MOUNT HOLLY.		
1790	West + 2° 24'	West + 0° 14'.54	2° 09'.416	
1795	" 2° 15'	" 0° 5'.17	2° 09'.832	
1800	" 2° 06'	East - 0° 04'.24	2° 10'.248	
1805	" 2° 06'	0° 04'.6	2° 10'.664	
1810	" 2° 06'	0° 05'	2° 11'.08	
1815	" 2° 11'.5 ^a	0° 0' ^b	2° 11'.5	* Subtract 0'.416
1820	" 2° 17'	West + 0° 05'.92	2° 11'.08*	from each differ-
1825	" 2° 29'.5	" 0° 18'.84	2° 10'.66	ence to obtain the
1830	" 2° 42'	" 0° 31'.75	2° 10'.248	next following
1835	" 3° 0'	" 0° 51'.17	2° 09'.832	down to 1845.
1840	" 3° 19'	" 1° 09'.5	2° 09'.416	
1845	" 3° 46'. ^a	" 1° 37'. ^b	2° 09'. [†]	
1850	" 4° 07'	" 1° 49'.88	2° 17'.12	† Add 1'.625 to
1855	" 4° 33'	" 2° 07'.76	2° 25'.24	each difference to
1860	" 4° 59'	" 2° 26'.62	2° 33'.37	get the difference
1865	" 5° 26'.5	" 2° 45'	2° 41'.5	for the succeeding
1870	" 5° 54'	" 3° 04'.38	2° 49'.62	year.
1875	" 6° 20'	" 3° 22'.25	2° 57'.75	
1877	" 6° 30'. ^a	" 3° 29'. ^c	3° 01'	
1880	" 6° 46'	" 3° 40'.12	3° 05'.875	

a. Interpolation from Assistant Schott's tables of decennial values of magnetic variation, U. S. Coast Survey Report for 1874.

b. From chart of Isogonic lines 1870. The positions of the curves with respect to Mount Holly were measured from additions in pen by Mr. Schott.

c. Observation by Beaton Smith.